

Pattern Occurrences in Random Walks

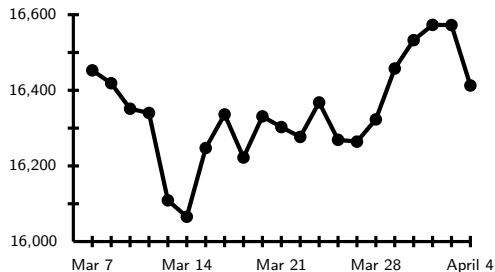
Sergi Elizalde Megan Martinez

Dartmouth College

Permutation Patterns, 2014

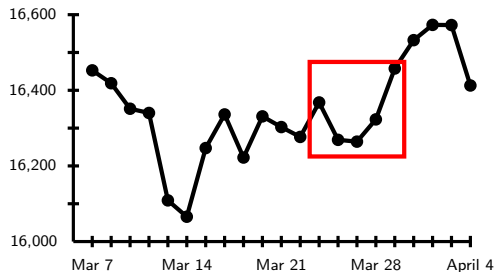
Patterns in Economics

Dow Jones Industrial Average: March 7, 2014 - April 4, 2014



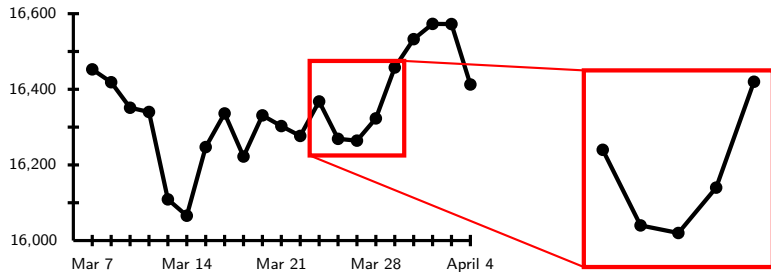
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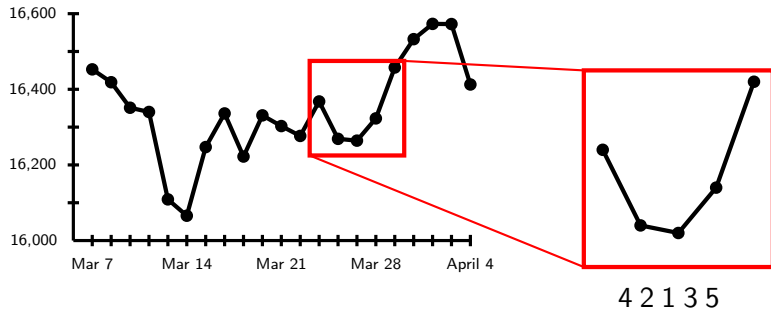
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 - ▶ At time i the walk is at $Z_i := X_1 + X_2 + X_3 + \dots + X_i$
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 - ▶ At time i the walk is at $Z_i := X_1 + X_2 + X_3 + \dots + X_i$
 - ▶ We call the X_i 's *steps*.
- ▶ We say the pattern $\pi \in S_k$ occurs in the random walk $Z_0, Z_1, \dots, Z_{n-1}$ if there exists some subsequence $Z_i, Z_{i+1}, \dots, Z_{i+k-1}$ that is order isomorphic to π .

Probabilities

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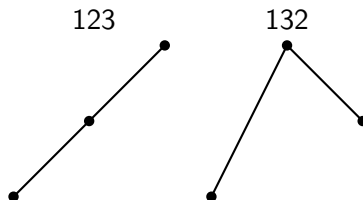
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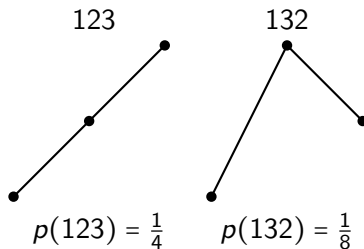
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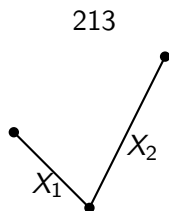
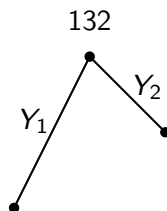
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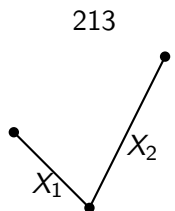


Probabilities (cont.)

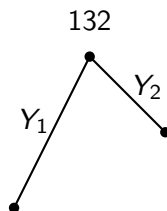
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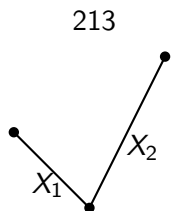


$$X_1 < 0$$
$$X_1 + X_2 > 0$$

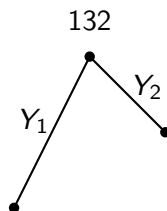


$$Y_2 < 0$$
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Probabilities (cont.)



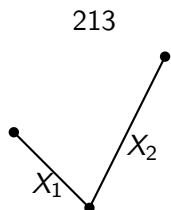
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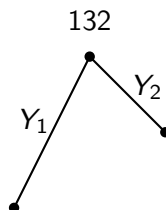
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- ▶ 213 and 132 have the same probability of occurring regardless of probability distribution

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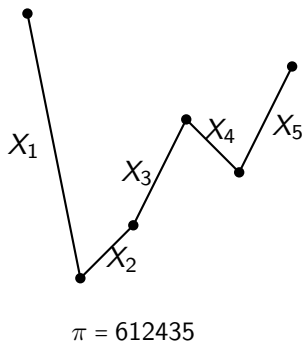


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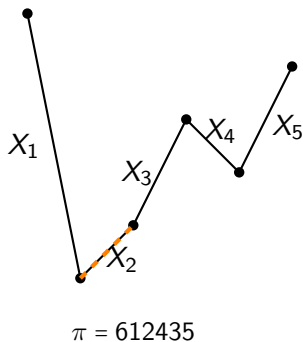
- ▶ 213 and 132 have the same probability of occurring regardless of probability distribution
- ▶ Question: Can we tell when two patterns have the same probability of occurring, regardless of distribution?

Encoding Patterns: Canonical Inequalities

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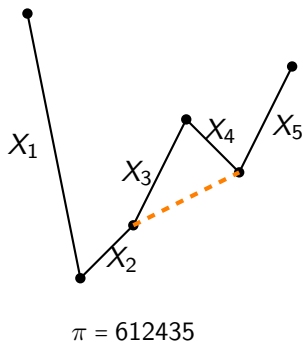


Encoding Patterns: Canonical Inequalities



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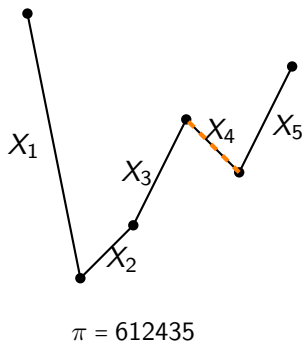
Encoding Patterns: Canonical Inequalities



$$X_2 > 0$$

$$X_3 + X_4 > 0$$

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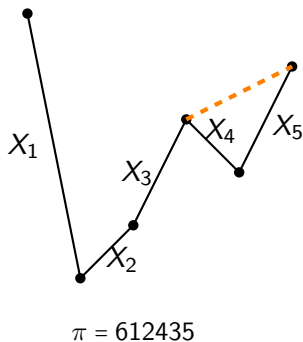


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Encoding Patterns: Canonical Inequalities



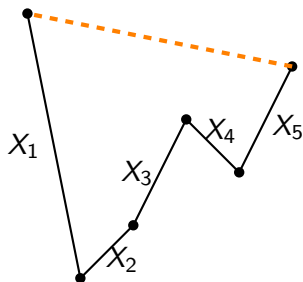
$$X_2 > 0$$

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$$X_4 < 0$$

$$X_4 + X_5 > 0$$

Encoding Patterns: Canonical Inequalities



$\pi = 612435$

$$X_2 > 0$$

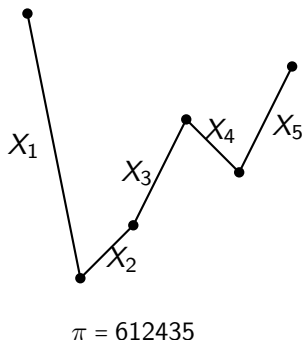
$$X_3 + X_4 > 0$$

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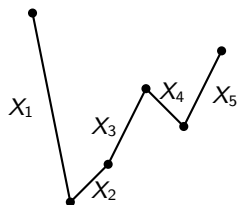
$$X_4 < 0$$

$$X_4 + X_5 > 0$$

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Canonical Inequalities for 612435

Equivalent Canonical Inequalities



$$\pi = 612435$$

$$X_2 > 0$$

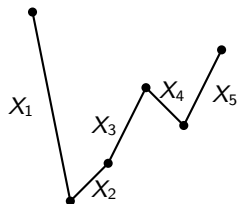
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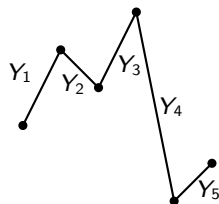
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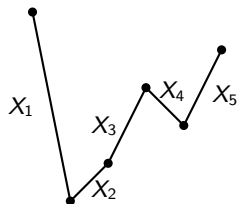
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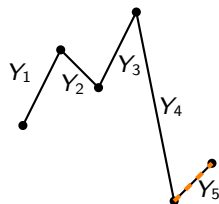
$$\tau = 354612$$

Equivalent Canonical Inequalities



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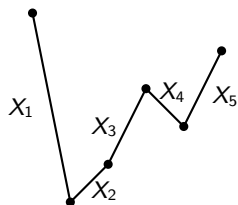
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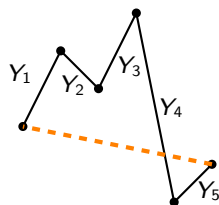
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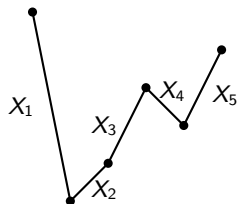
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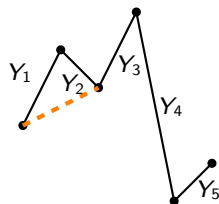
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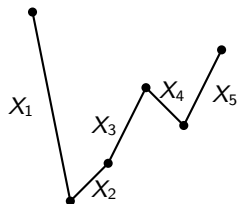
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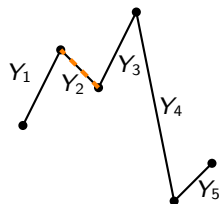
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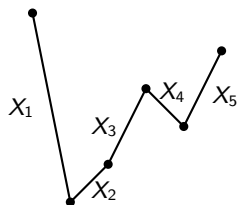
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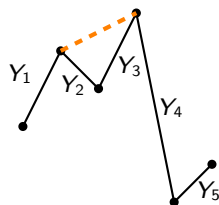
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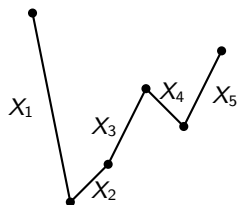
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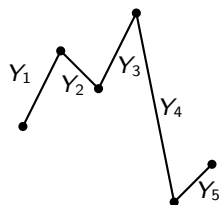
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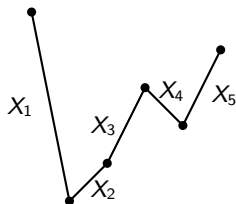
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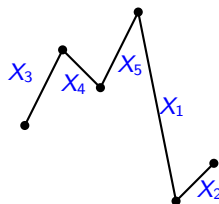
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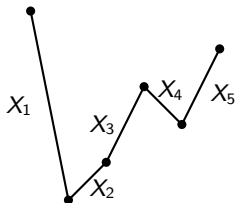


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The canonical inequalities for π and τ are **equivalent up to permutation of the indices**.

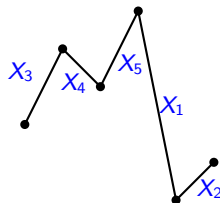
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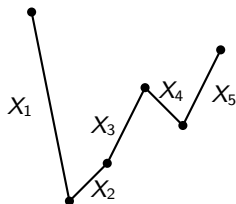


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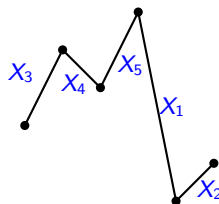
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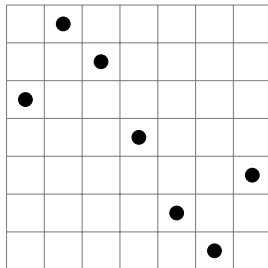
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- ▶ Fact: $\pi^{RC} \sim \pi$
- ▶ Can we characterize the equivalence classes?

Patterns on Grids

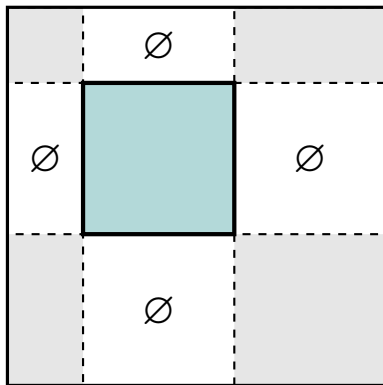
5764213 $\mapsto$



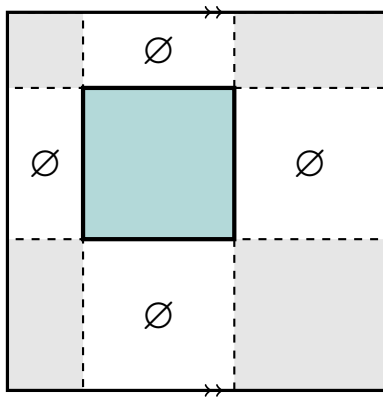
- Fill cell $(i, \pi(i))$ for all $i \in [n]$.

Cylindrical Blocks

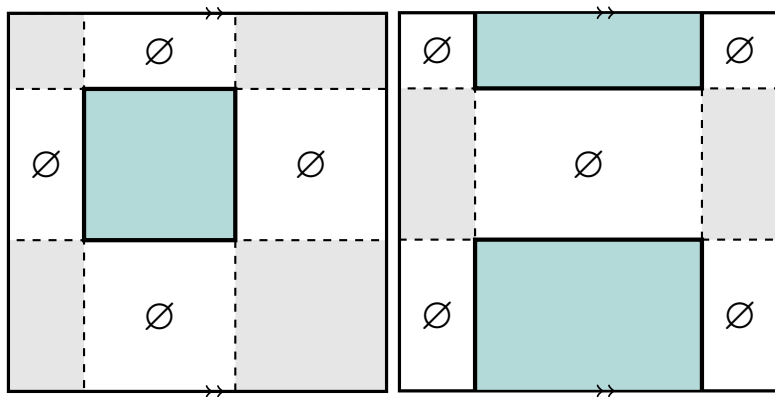
Cylindrical Blocks



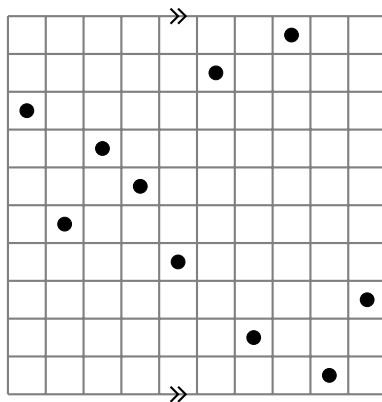
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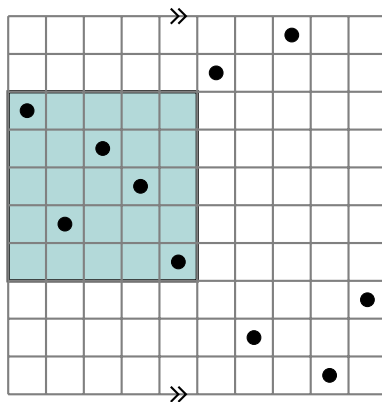


Examples of Cylindrical Blocks



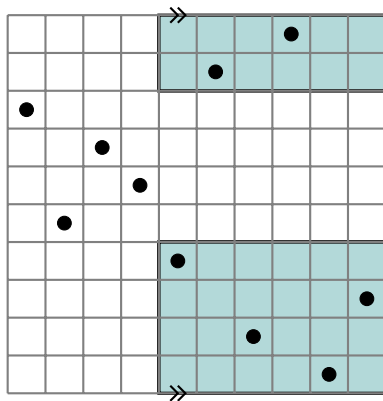
8,5,7,6,4,9,2,10,1,3

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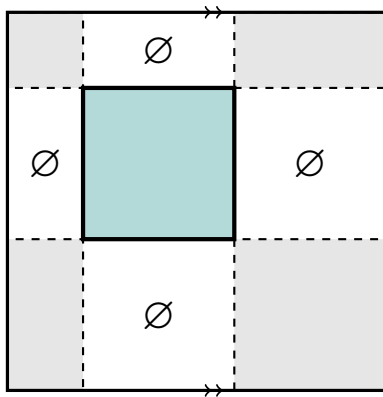
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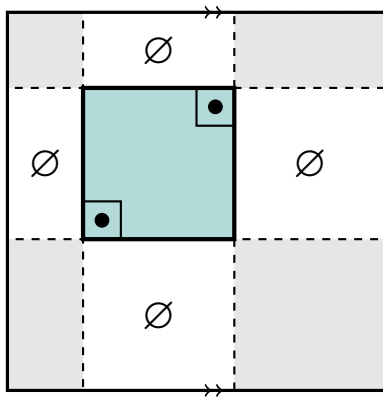


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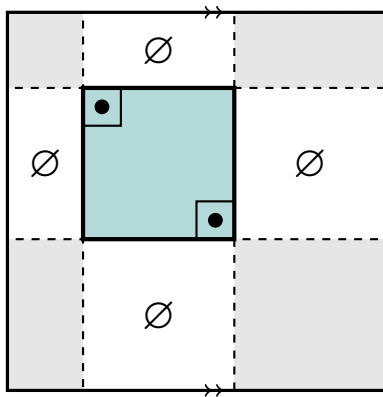
Bordered Blocks



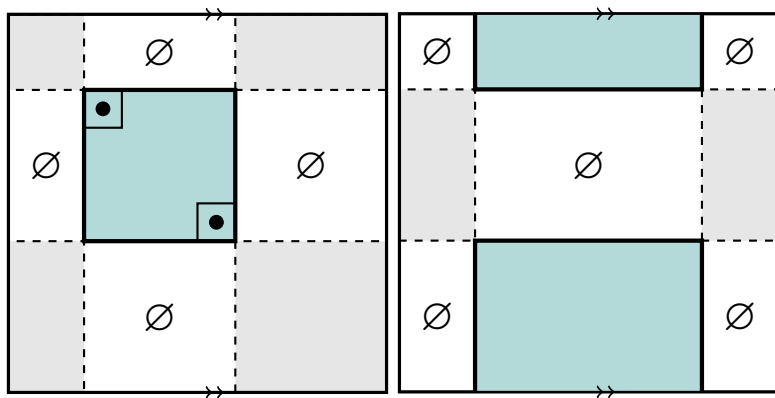
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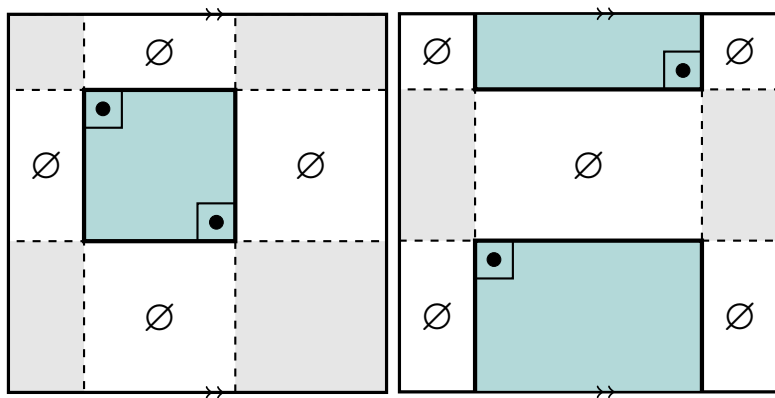
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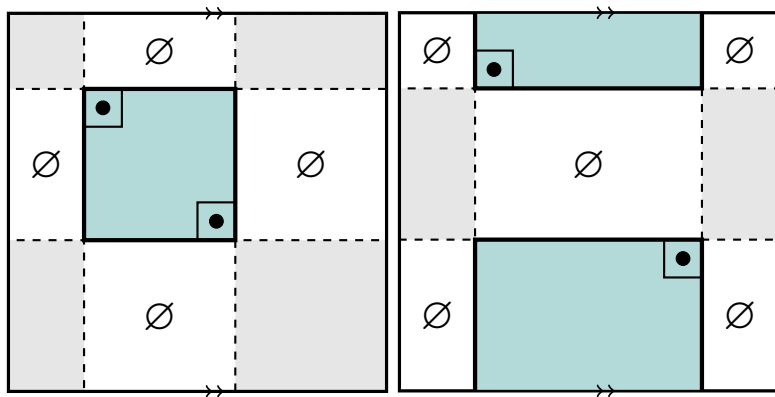
Bordered Blocks



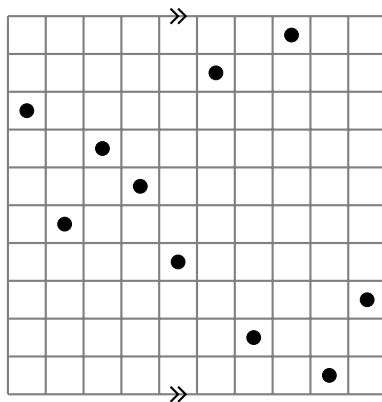
Bordered Blocks



Bordered Blocks

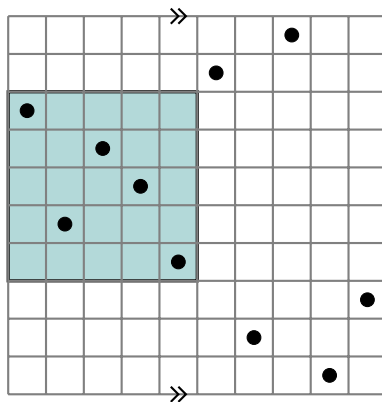


Examples of Bordered Cylindrical Blocks



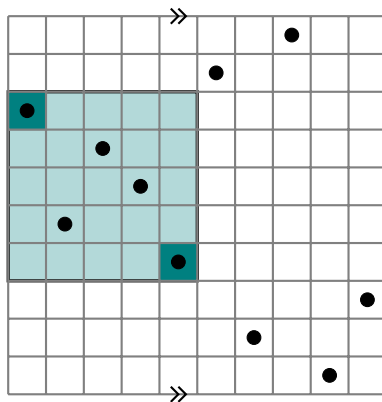
8,5,7,6,4,9,2,10,1,3

Examples of Bordered Cylindrical Blocks



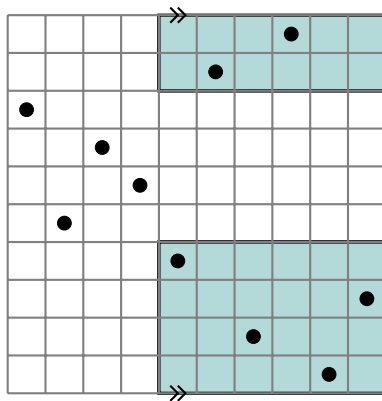
8,5,7,6,4,9,2,10,1,3

Examples of Bordered Cylindrical Blocks



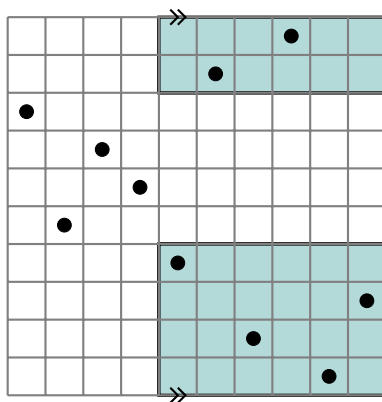
8,5,7,6,4,9,2,10,1,3

Examples of Bordered Cylindrical Blocks



8,5,7,6,4,9,2,10,1,3

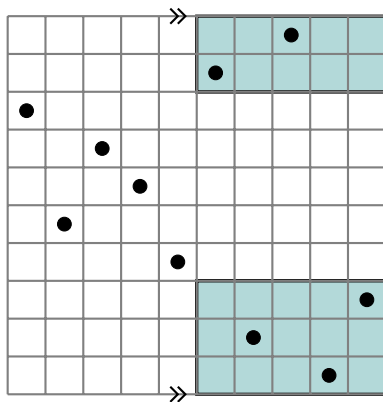
Examples of Bordered Cylindrical Blocks



8,5,7,6,4,9,2,10,1,3

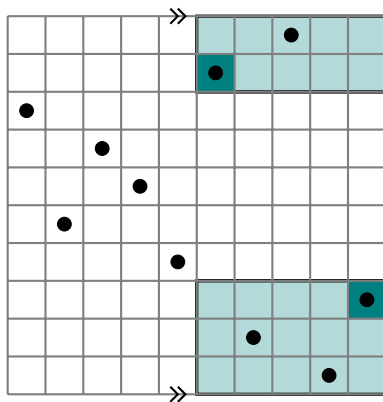
Not bordered!

Examples of Bordered Cylindrical Blocks



8,5,7,6,4,9,2,10,1,3

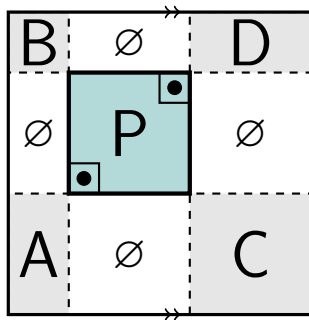
Examples of Bordered Cylindrical Blocks



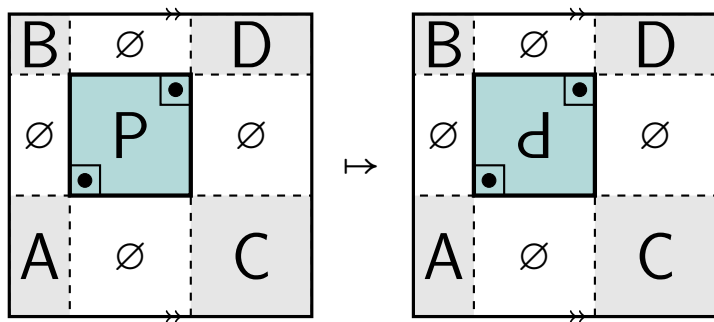
8,5,7,6,4,9,2,10,1,3

Flipping Bordered Cylindrical Blocks

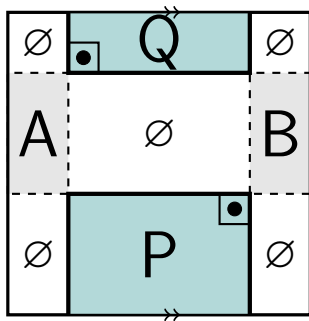
Flipping Bordered Cylindrical Blocks



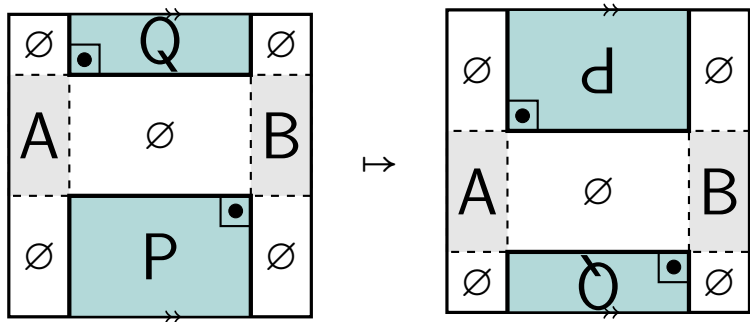
Flipping Bordered Cylindrical Blocks



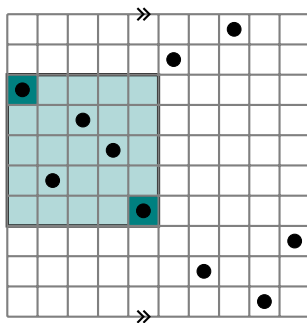
Flipping Bordered Cylindrical Blocks



Flipping Bordered Cylindrical Blocks

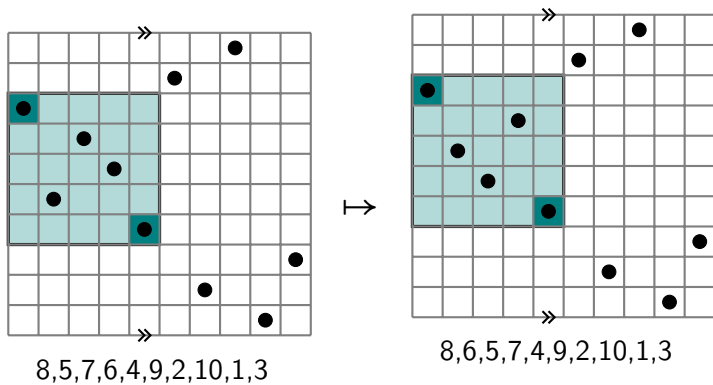


Flipping Bordered Cylindrical Blocks

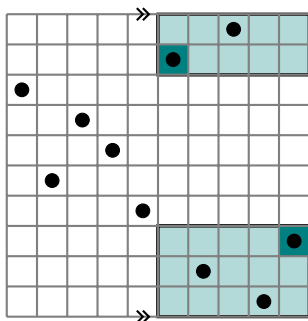


8,5,7,6,4,9,2,10,1,3

Flipping Bordered Cylindrical Blocks

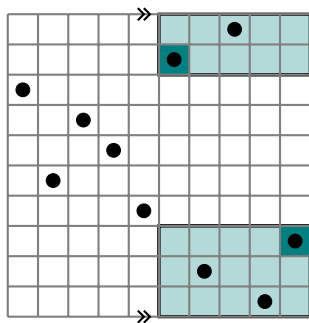


Flipping Bordered Cylindrical Blocks

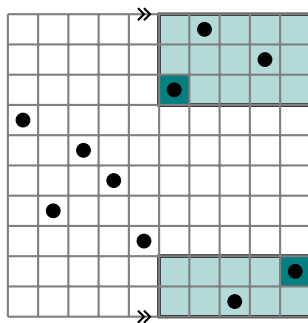


8,5,7,6,4,9,2,10,1,3

Flipping Bordered Cylindrical Blocks



8,5,7,6,4,9,2,10,1,3



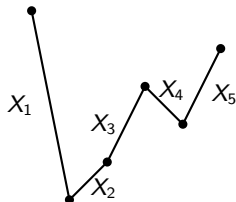
7,4,6,5,3,8,10,1,9,2

Characterizing Equivalence Classes

Theorem

Given $\pi, \tau \in S_n$, $\tau \sim \pi$ if and only if τ can be obtained from π by a sequence of flips of bordered cylindrical blocks and the reverse-complement operation.

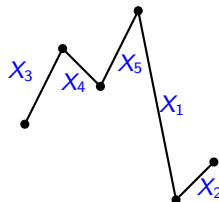
Recall



$$\pi = 612435$$

$$\begin{aligned} X_2 &> 0 \\ X_3 + X_4 &> 0 \\ X_4 &< 0 \\ X_4 + X_5 &> 0 \\ X_1 + X_2 + X_3 + X_4 + X_5 &< 0 \end{aligned}$$

$X_1 \leftrightarrow Y_4$
$X_2 \leftrightarrow Y_5$
$X_3 \leftrightarrow Y_1$
$X_4 \leftrightarrow Y_2$
$X_5 \leftrightarrow Y_3$



$$\tau = 354612$$

$$\begin{aligned} X_2 &> 0 \\ X_3 + X_4 &> 0 \\ X_4 &< 0 \\ X_4 + X_5 &> 0 \\ X_1 + X_2 + X_3 + X_4 + X_5 &< 0 \end{aligned}$$

Fact: $\tau \sim \pi$.

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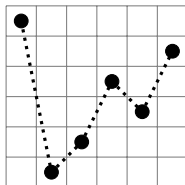
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Example

Show: $613245 \sim 612435$



$\pi = 612435$

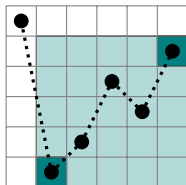
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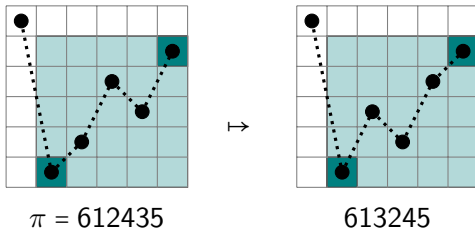
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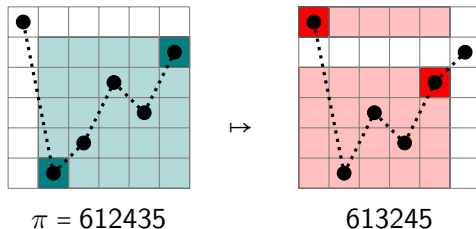
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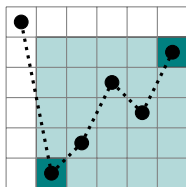
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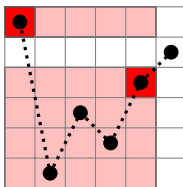
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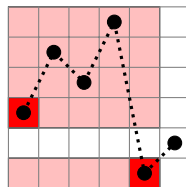
Show: $613245 \sim 612435$



$\pi = 612435$



613245



$\tau = 354612$

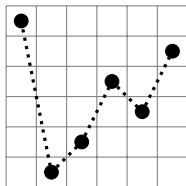
Characterizing Equivalence Classes

Theorem

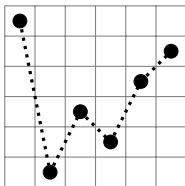
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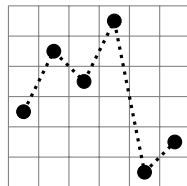
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613245



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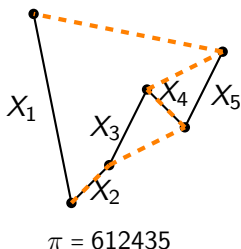
Idea of the Proof

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- ▶ Consider all permutations of the variables in the canonical inequalities.
- ▶ When is the result a set of canonical inequalities?

Idea of the Proof

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- ▶ When is the result a set of canonical inequalities?



$$\begin{aligned}X_2 &> 0 \\X_3 + X_4 &> 0 \\X_4 &< 0 \\X_4 + X_5 &> 0 \\X_1 + X_2 + X_3 + X_4 + X_5 &< 0\end{aligned}$$

- ▶ The orange path completely describes the inequalities.

Edge Diagram

Edge Diagram

$$X_2 > 0$$

$$X_3 + X_4 > 0$$

$$X_4 < 0$$

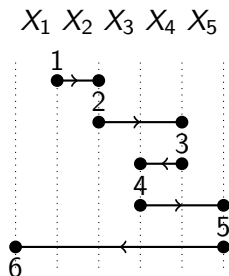
$$X_4 + X_5 > 0$$

$$X_1 + X_2 + X_3 + X_4 + X_5 < 0$$

Edge Diagram

$$\begin{aligned} X_2 &> 0 \\ X_3 + X_4 &> 0 \\ X_4 &< 0 \\ X_4 + X_5 &> 0 \\ X_1 + X_2 + X_3 + X_4 + X_5 &< 0 \end{aligned}$$

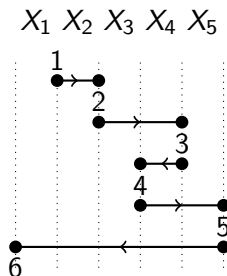
$\mapsto$



Edge Diagram for 612435

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$$\begin{aligned} X_2 &> 0 \\ X_3 + X_4 &> 0 \\ X_4 &< 0 \\ X_4 + X_5 &> 0 \\ X_1 + X_2 + X_3 + X_4 + X_5 &< 0 \end{aligned} \quad \mapsto$$



Edge Diagram for 612435

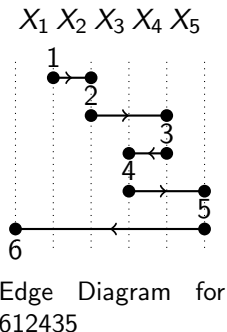
- ▶ One inequality $\Rightarrow$ One edge
- ▶ Direction of inequality $\Rightarrow$ Direction on edge
- ▶ An edge diagram that forms a path that visits all the vertices corresponds to a permutation. Call these edge diagrams **valid**.

Permuting Steps in an Edge Diagram

- ▶ Considering all permutations of variables in canonical inequalities

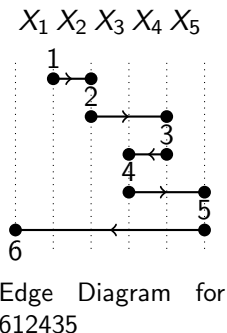
Permuting Steps in an Edge Diagram

- ▶ Considering all permutations of variables in canonical inequalities



Permuting Steps in an Edge Diagram

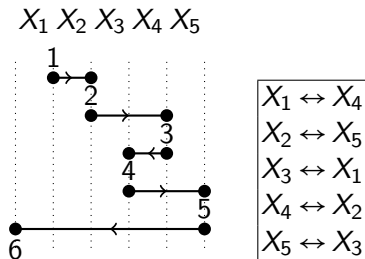
- ▶ Considering all permutations of variables in canonical inequalities



- ▶ When permuting the steps in an edge diagram:
 - ▶ Permute steps so edges remain intact
 - ▶ Direction on edges do not change
 - ▶ Resulting edge diagram should form a path

Permuting Steps in an Edge Diagram

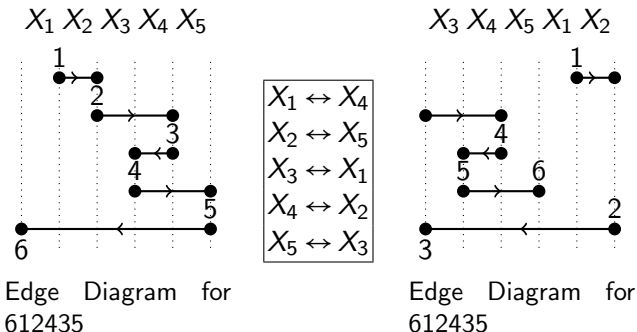
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Permuting Steps in an Edge Diagram

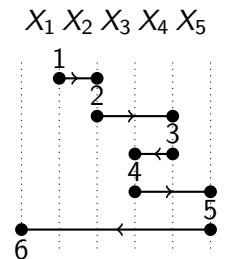
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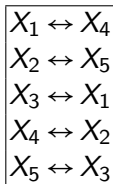
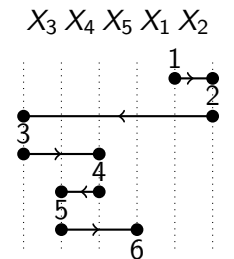
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Edge Diagram for
612435

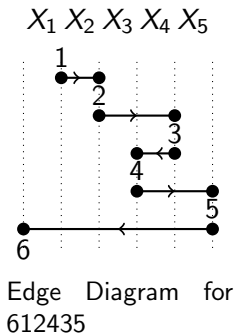


Edge Diagram for
354612

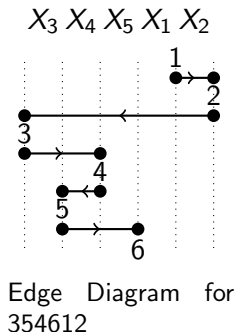
- When permuting the steps in an edge diagram:
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Permuting Steps in an Edge Diagram

- Considering all permutations of variables in canonical inequalities



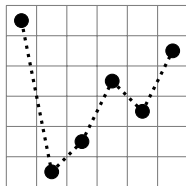
$$\begin{array}{l} X_1 \leftrightarrow X_4 \\ X_2 \leftrightarrow X_5 \\ X_3 \leftrightarrow X_1 \\ X_4 \leftrightarrow X_2 \\ X_5 \leftrightarrow X_3 \end{array}$$



- $\{X_3, X_4, X_5\}$ will always be consecutive
- A consecutive set of steps is called **cohesive** if those steps must be consecutive after a valid permutation is applied on the edge diagram (among other conditions).

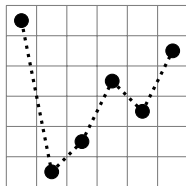
Bordered Cylindrical Blocks vs. Valid Flips

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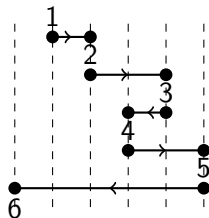
$$\pi = 612435$$

Bordered Cylindrical Blocks vs. Valid Flips

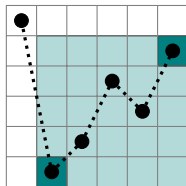


$$\pi = 612435$$

$X_1 \ X_2 \ X_3 \ X_4 \ X_5$

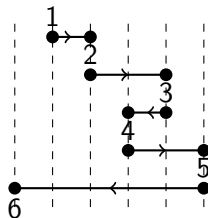


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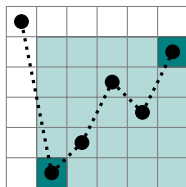


$$\pi = 612435$$

$X_1 \ X_2 \ X_3 \ X_4 \ X_5$

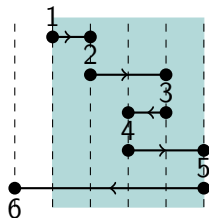


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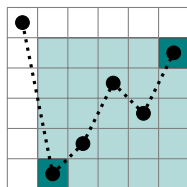


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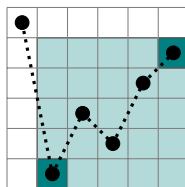
$X_1 \ X_2 \ X_3 \ X_4 \ X_5$



Bordered Cylindrical Blocks vs. Valid Flips



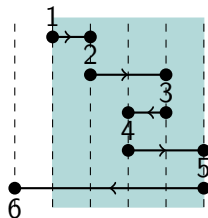
$\mapsto$



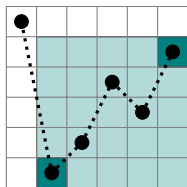
$\pi = 612435$

613245

$X_1 \ X_2 \ X_3 \ X_4 \ X_5$

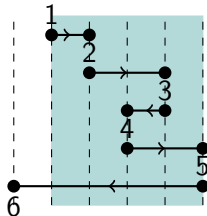
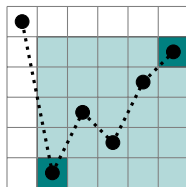


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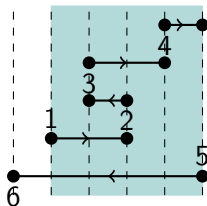
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$X_1 \ X_2 \ X_3 \ X_4 \ X_5$

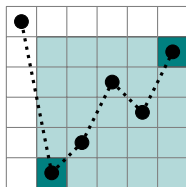


613245

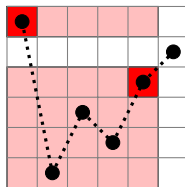
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$\mapsto$

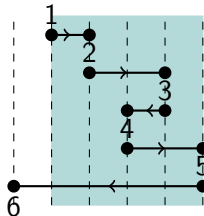


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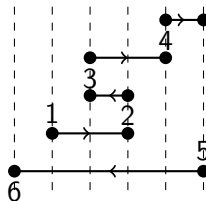
613245

$X_1 \ X_2 \ X_3 \ X_4 \ X_5$

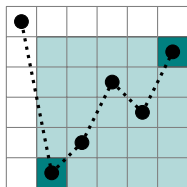
$X_1 \ X_2 \ X_3 \ X_4 \ X_5$



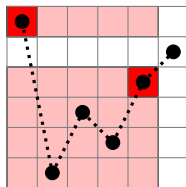
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$\mapsto$

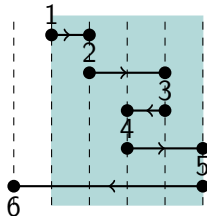


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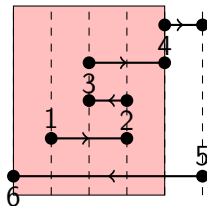
613245

$X_1 \ X_2 \ X_3 \ X_4 \ X_5$

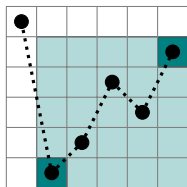
$X_1 \ X_2 \ X_3 \ X_4 \ X_5$



$\mapsto$

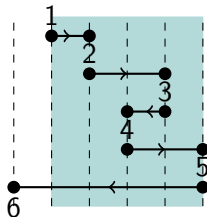
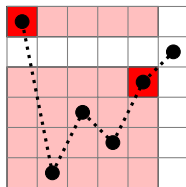


Bordered Cylindrical Blocks vs. Valid Flips



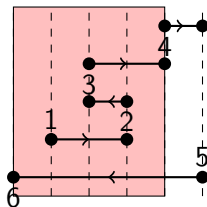
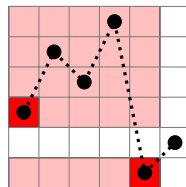
$$\pi = 612435$$

$X_1 \ X_2 \ X_3 \ X_4 \ X_5$



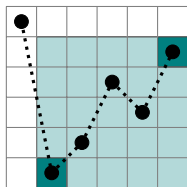
$$613245$$

$X_1 \ X_2 \ X_3 \ X_4 \ X_5$



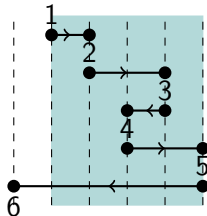
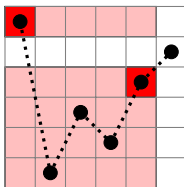
$$\tau = 354612$$

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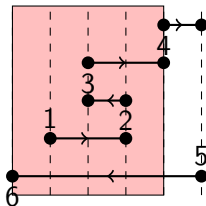
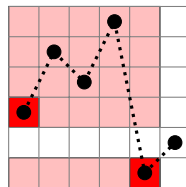
$\pi = 612435$

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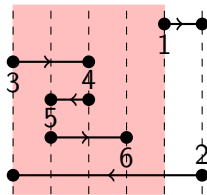
613245

$X_1 \ X_2 \ X_3 \ X_4 \ X_5$



$\tau = 354612$

$X_1 \ X_2 \ X_3 \ X_4 \ X_5$



The Theorem Rephrased

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- ▶ A **valid flip** is a consecutive set of steps that can be flipped and yield an edge diagram corresponding to a permutation.

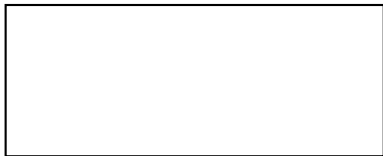
The Theorem Rephrased

- ▶ A **valid flip** is a consecutive set of steps that can be flipped and yield an edge diagram corresponding to a permutation.
- ▶ Theorem Rephrased: For $\tau, \pi \in S_n$, $\tau \sim \pi$ if and only if the edge diagram of τ can be obtained from the edge diagram of π with a sequence of valid flips.

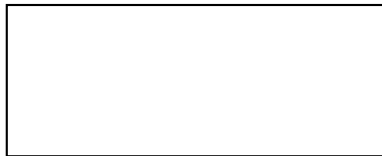
The Argument

The Argument

- ▶ Start with $\tau \sim \pi$



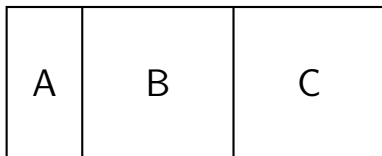
Edge Diagram for π



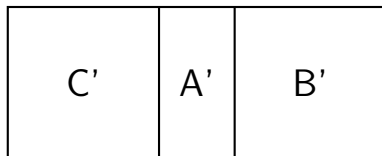
Edge Diagram for τ

The Argument

- ▶ Start with $\tau \sim \pi$
- ▶ Partition edge diagrams of π and τ maximally (and non-trivially) into cohesive blocks.



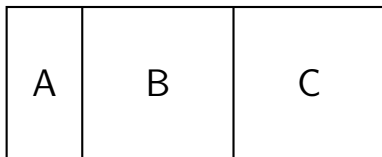
Edge Diagram for π



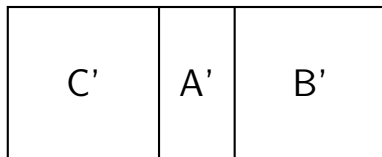
Edge Diagram for τ

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- ▶ Start with $\tau \sim \pi$
- ▶ Partition edge diagrams of π and τ maximally (and non-trivially) into cohesive blocks.
 - ▶ Cohesive blocks will correspond between π and τ .



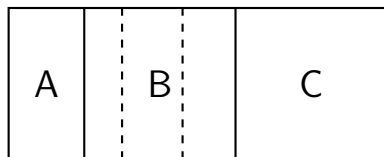
Edge Diagram for π



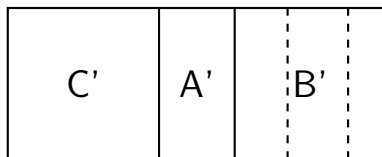
Edge Diagram for τ

The Argument

- ▶ Start with $\tau \sim \pi$
- ▶ Partition edge diagrams of π and τ maximally (and non-trivially) into cohesive blocks.
 - ▶ Cohesive blocks will correspond between π and τ .
- ▶ Each block of the partition is then partitioned in a similar way.
- ▶ These nested partitions on π and τ will correspond to each other.
- ▶ Induction argument used on each individual partition.



Edge Diagram for π



Edge Diagram for τ

Future Work

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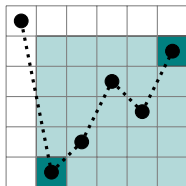
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- ▶ How many equivalence classes are there in S_n ?
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- ▶ What is the maximal size of an equivalence class in S_n ?

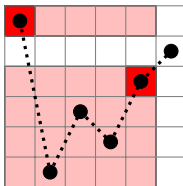
Future Work

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- ▶ How many equivalence classes are there in S_n ?
- ▶ Given $\pi \sim \tau$ find an algorithm to construct a sequence of valid flips.
- ▶ What is the maximal size of an equivalence class in S_n ?
- ▶ Can we put a partial ordering on the equivalence classes (by probability of occurrence regardless of distribution)? What would this look like?

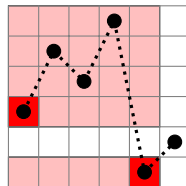
Thank you!



$\pi = 612435$



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$\tau = 354612$